Calculus was my third worst exam, so these are quite possibly wrong.

1ai: Least upper bound is .

We have for all S =

1

2

,

2

3

,

3

4

, . . . ,

n

n + 1

, . . .

. so , so is an upper bound.

Suppose there exists an upper bound of such that . Take , then and so , therefore , which means cannot be an upper bound. This is a contradiction so there is no upper bound of that is less than . Hence is the least upper bound of .

1aii: Infimum is , minimum is and the maximum does not exist.

Let alone do the paper in 80 mins, I’m not gonna be able to copy my answers into here in 80 mins

1bi: Let be the sequence given by . We prove that for all there exists an such that for all we have , where . We have iff iff iff iff iff . So let and thus we have that .

1bii:

1biii: Let and . Then . When is odd, and when is even, . So clearly , so by the sandwich theorem we have .

1ci: This question seems like it’s been put there to insult your intelligence. Not sure if you can just do it the easy way and say it’s a constant function. Anyway, here’s the long version:

As per the definition of continuity at a point, we show that , i.e. for all there exists a such that whenever we have that . Assume , then we have

, which is true for all by definition of . Assume instead that . Then we have which is again true for all . So pick any and you’ll be fine, hence it’s continuous.

1cii: This one says “prove” so I’m pretty sure the long way is the only acceptable way. We prove that for all there exists a such that for all , whenever , we have that. Same sort of thing as always:

So let , which doesn’t depend on or , meaning that is uniformly continuous on .

1di: D’Alembert limit ratio test.

.

We have so , thus converges.

1dii: Proof by obvious.

by

geometric series. This might not be what they were looking for.

1diii: The notes say that if a series converges then its sequence of summands converges to . The contrapositive of this is that if a sequence does not converge to then the corresponding series does not converge. Clearly so the sequence of summands does not converge to . Hence the series does not converge (and so it diverges).

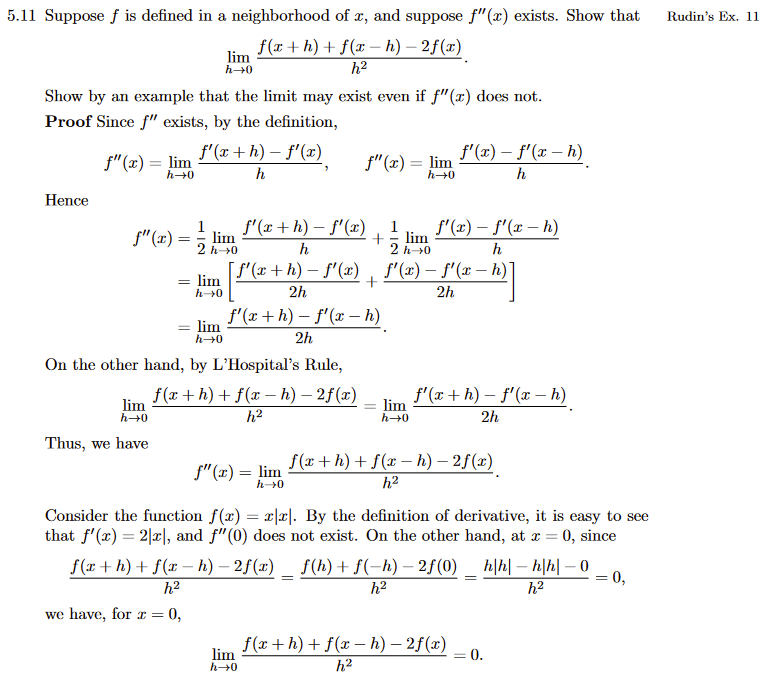
During the open book exam many people made the shocking discovery that some of the questions in part 2 could be found online, so some of these are just links to the internet (screenshots provided in case one of these pages go down/link breaks)

2ai: Clearly we assume . By direct substitution with the definition, we have:

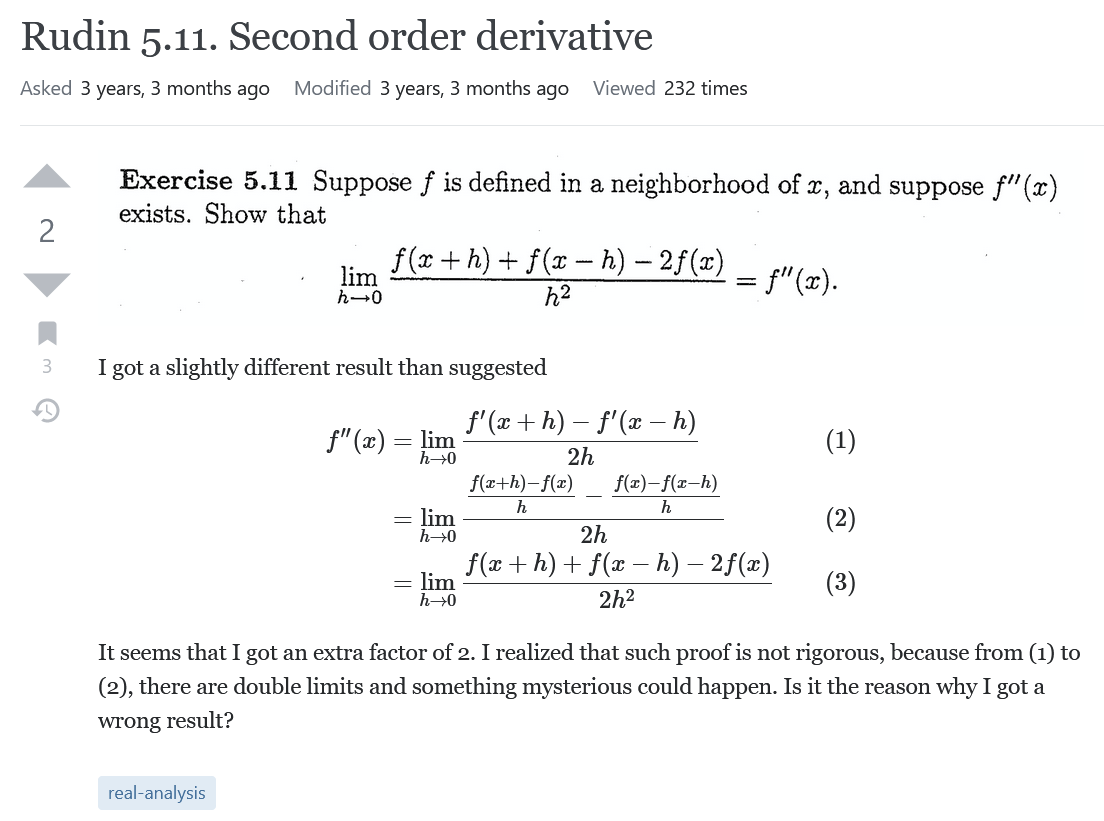
Note the numerator is constant and because by simple addition of limits. As , we have that

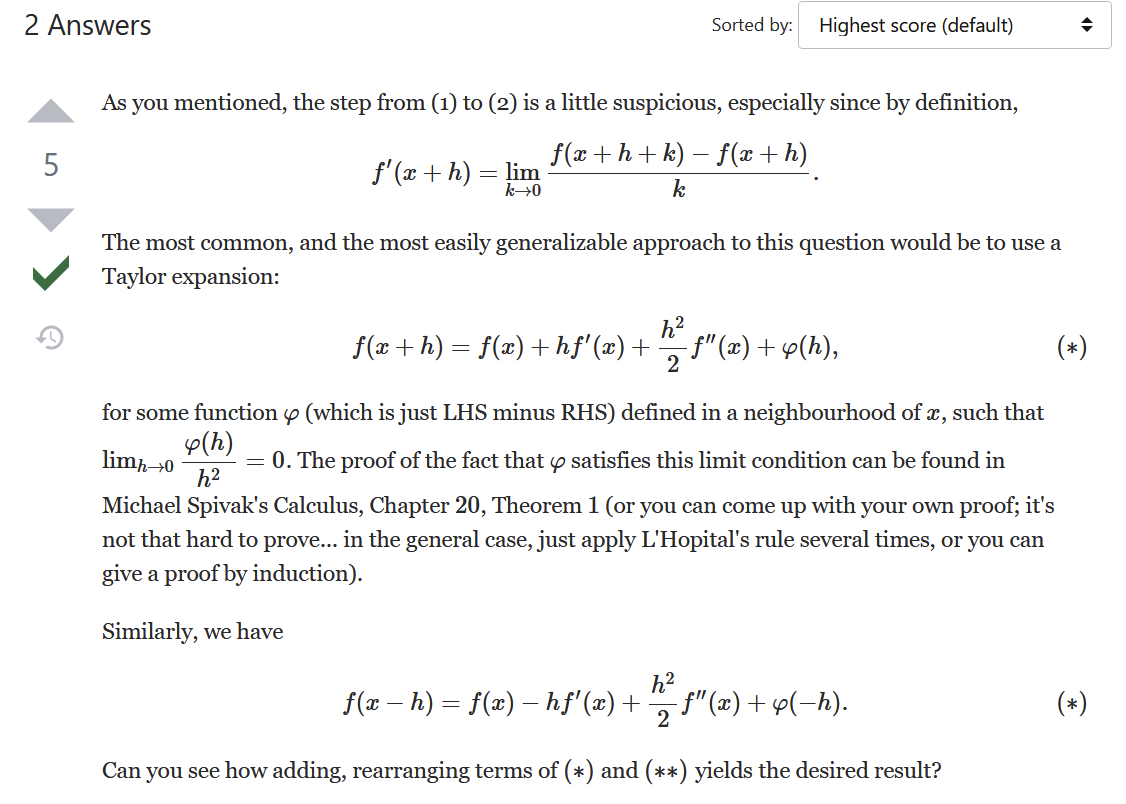
Due to the algebra of limits.

2aii: Rudin 5.11 <https://www.math.hkust.edu.hk/~majhu/Math203/Rudin/Homework20.pdf>



<https://math.stackexchange.com/q/3268204>





2bi: Using the given fact, we get

()

Using the fact:

2bii: Note that

 (can’t find a way to do the large { )

Also, note that

Thus

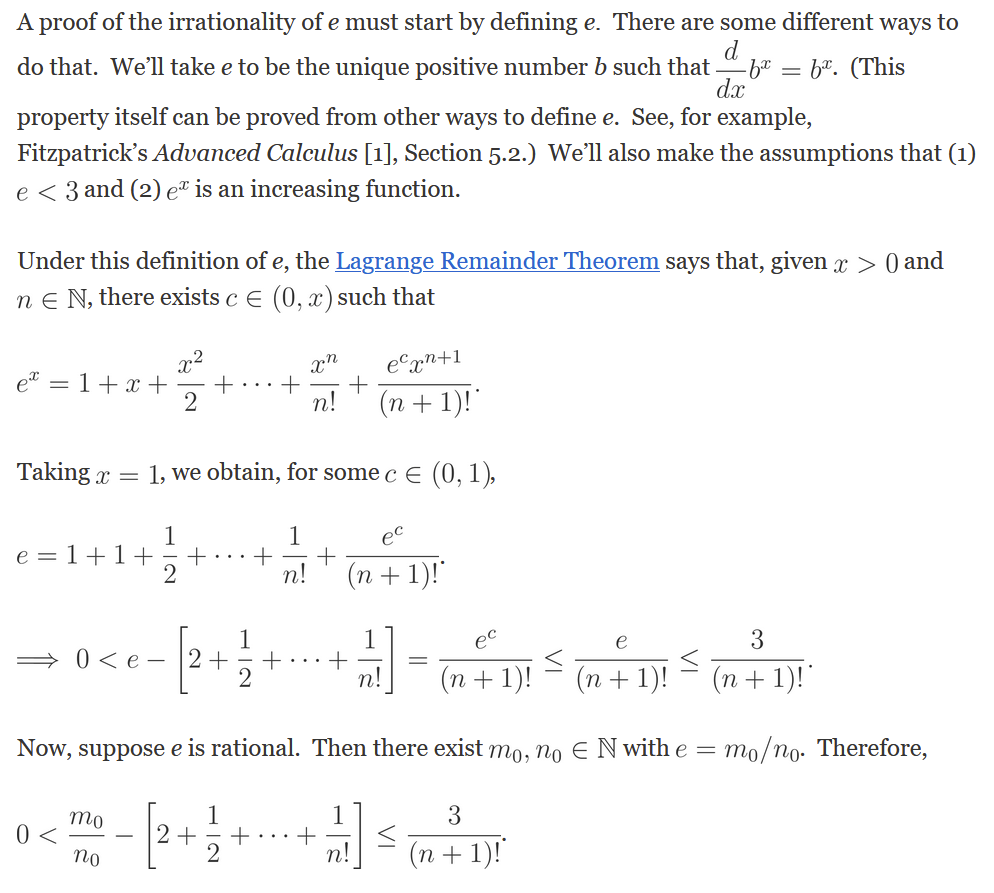
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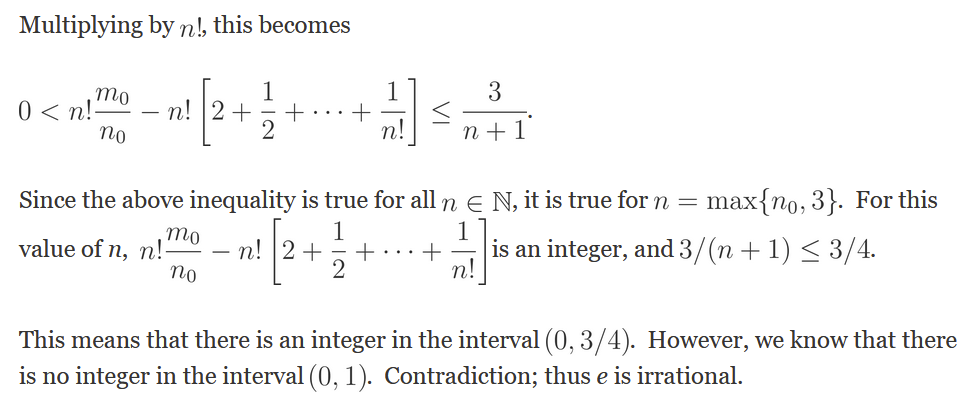
()()

2ci: Recall that can be represented by its power series . Therefore we have that the equation simplifies into

2cii: <https://mikespivey.wordpress.com/2014/12/15/proof-of-the-irrationality-of-e/>

(no idea how you can be expected to produce this proof in an exam without using the internet lol)



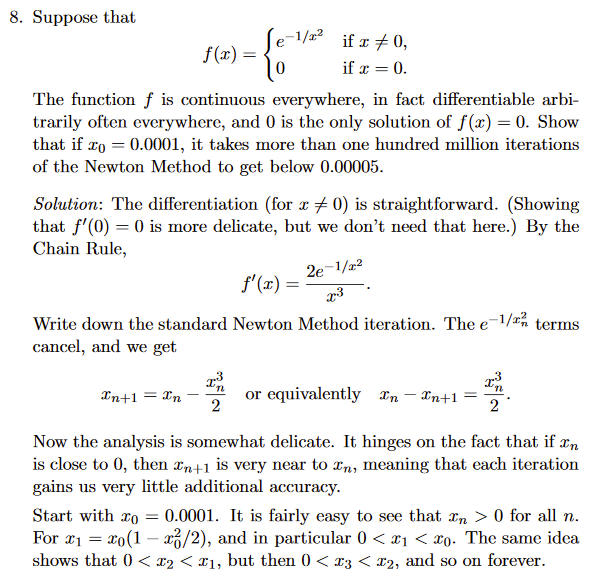


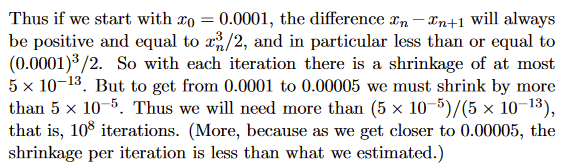
2d:

UBC math 104

<https://personal.math.ubc.ca/~anstee/math104/104newtonmethod.pdf>

<https://personal.math.ubc.ca/~anstee/math104/104newton-solution.pdf> q8





2ei (Jyry’s version): We go property-by-property

**Positive:** This is trivially true by definition of as its domain is wholly positive.

**Symmetric:** Let :

()()()()

**Distance from point to itself is zero:** Let . Then

()()()

By definition of

**Triangle inequality:** Let . Then

()()()()()

As is monotonically increasing, we have that f preserves ordering of the reals (i.e. ()()). As () is a metric on , we have

Hence

()()()()()

With all these properties, is a metric on

2eii (Jyry’s version): from part 2ei, it suffices to show that is concave. Its domain already fits the function in part ei as we are dealing with positive reals. Therefore we want to show only that [):

Note that [). Then

()

As is a monotonically increasing function, we have

()

Thus we can use part ei and we are done

2e (internet version): <https://math.stackexchange.com/a/707533>

